SCSM for Calculation of Motion-Induced Eddy Currents in Isotropic and Anisotropic Conductive Objects

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A new approach to determine the defect response signals from defects located in conducting objects of any size based on the surface charge simulation method is presented in the framework of the Lorentz force eddy current testing. The results are verified by FEM simulations and will be validated by measurements.

***Index Terms*—Eddy currents, finite element analysis, integral equations, nondestructive testing, permanent magnets, surface charges.**

I. Introduction

ELECTRODYNAMICS of moving media is one of fundamental problems in theoretical electromagnetics. There is an essential need for efficient computation methods in a variety of applications such as magnetic levitation, eddy currents brakes, or electromagnetic damping and coupling. The principle of motional induction is also utilized in the framework of nondestructive testing (NDT) [1]. Methods based on this principle are classified as motion-induced eddy current testing (MECT) methods. One associated MECT method is called Lorentz force eddy current testing (LET) [2]. In LET, the object under test moves relative to a permanent magnet system, which produces a stationary magnetic field. Its basic principle is shown in Fig. 1. The interaction between the induced eddy currents and the magnetic field results in a Lorentz force, which acts on both the specimen and the magnet system itself. In the presence of a defect the induced eddy currents and hence the force signal are perturbed. The LET method involves distinct line and surface scans of the object under test. The prediction of the Lorentz force profiles necessitates the application of numerical methods. The bigger the area to be evaluated, the more simulations have to be conducted. Consequently, most efficient routines are needed to determine the so-called forward problem in view of defect reconstruction. Recent optimization studies showed that rather complicated magnet systems (see Fig. 1), which include highly saturating ferromagnetic materials such as iron-cobalt alloys may be advantageous compared to standard magnet geometries. However, the implementation of nonlinear magnetic material significantly increases the computational effort needed to determine the force profiles numerically.

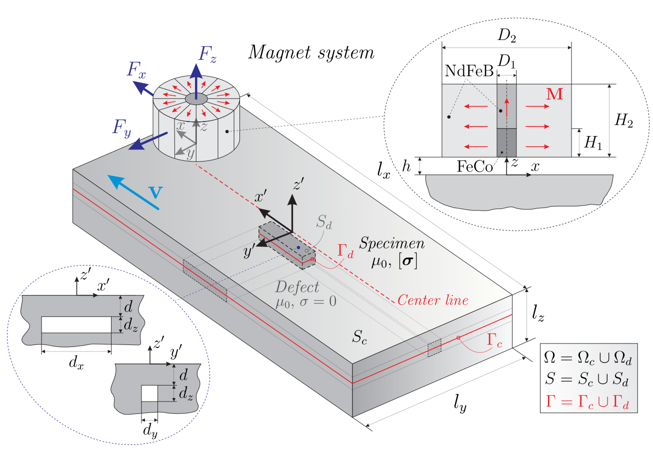

Fig. 1. Principle of Lorentz force eddy current testing.

In this paper, an approach based on the principles of the surface charge simulation method (SCSM) [3] is presented. The field problem in the SCSM is described by surface integrals of the Fredholm type of the first kind. The coefficients of the distributed surface charges situated on the boundaries are determined by fulfilling the imposed boundary conditions. The SCSM is well known [4] but its application to electrodynamics of moving media is only merely studied [5]. Owing the recent developments in magnet optimization, we consider the approach which is independent on the applied magnet system. Particular attention is devoted to proof the validity of the proposed approach by comparing the SCSM results to data obtained by FEM. Additionally, the results are compared to experimental data demonstrating the practical applicability and accuracy of the proposed approach.

II. Description of the Method

The analysis of the LET system shown in Fig. 1 is performed in the coordinate system attached to the magnet system (PM). The state of the system is defined by vector fields $\mathbf{B}$ and $\mathbf{E}$ under assumption that the velocity of the moving conducting object is much less than the speed of light. Additionally, it is assumed that the magnetic field associated with eddy currents induced in the moving conductor is negligible in relation to the imposed primary magnetic field produced by the PM (*weak reaction approach* - WRA). This assumption is valid, for example, for conductive objects made of aluminum and moving with a speed of less than 0.5 m/s.

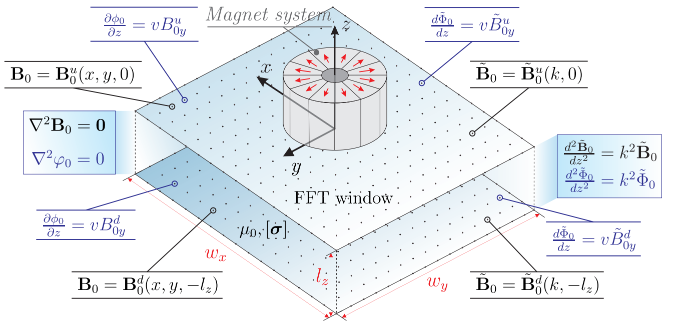

Fig. 2. Setup for the magnetic field $\mathbf{B}_0$ and the potential φ_0 calculations.

Applying the WRA, eddy currents induced in a conductor moving with the velocity $\mathbf{v}$ are determined from Ohm's law

$$\mathbf{J} = [\boldsymbol{\sigma}](\mathbf{E} + \mathbf{v} \times \mathbf{B}_0), \tag{1}$$

where $[\boldsymbol{\sigma}] = \mathrm{diag}(\sigma_{xx}, \sigma_{yy}, \sigma_{zz})$ is the electrical conductivity tensor and $\mathbf{B}_0$ is the primary magnetic flux density produced by the PM. The electric field $\mathbf{E}$ is expressed by the scalar electric potential φ as $\mathbf{E} = -\nabla\varphi$. In the SCSM, the potential φ at any point $\mathbf{r}$ in the conductor of isotropic conductivity ($\sigma_{ii} = \sigma_0$) is determined from

$$\varphi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int_S \frac{\kappa(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} \mathrm{d}S \tag{2}$$

where $\kappa(\mathbf{r}')$ is a surface charge density distributed over the surface S (Fig. 1). For conductors characterized by anisotropic conductivity ($\sigma_{xx} = \sigma_{yy} = \sigma_0$, $\sigma_{zz} = 0$), the following expression is used

$$\varphi(\mathbf{r}) = \frac{1}{2\pi\varepsilon_0} \int_\Gamma \tau(\mathbf{r}') \ln \frac{1}{|\mathbf{r}-\mathbf{r}'|} \mathrm{d}s + \varphi_0, \tag{3}$$

where $\tau(\mathbf{r}')$ is a line charge density on the boundary line Γ constructed as the intersection of surface S with the $X0Y$-plane cutting centrally the defect (Fig. 1). The φ_0 denotes the scalar electric potential in the infinitely large plate of thickness l_z without defect. To find eddy currents (1), the set of N SCSM equations is formulated using the condition $\mathbf{n} \cdot \mathbf{J}|_S = 0$ as

$$\frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \kappa_i \int_{S_i} \frac{\mathbf{n}_j \cdot (\mathbf{r}_j - \mathbf{r}'_i)}{|\mathbf{r}_j - \mathbf{r}'_i|^3} \mathrm{d}S_i = -\mathbf{n}_j \cdot (\mathbf{v} \times \mathbf{B}_{0j}), \tag{4}$$

for the isotropic case ($\kappa_i = \text{const on } S_i$, $S = \bigcup_{i=1}^N S_i$) and

$$\frac{1}{2\pi\varepsilon_0} \sum_{i=1}^{N} \tau_i \int_{\Gamma_i} \frac{\mathbf{n}_j \cdot (\mathbf{r}_j - \mathbf{r}'_i)}{|\mathbf{r}_j - \mathbf{r}'_i|^2} \mathrm{d}s_i = \mathbf{n}_j \cdot (\nabla\varphi_{0j} - \mathbf{v} \times \mathbf{B}_{0j}), \tag{5}$$

for the anisotropic one ($\tau_i = \text{const on } \Gamma_i$, $\Gamma = \bigcup_{i=1}^N \Gamma_i$). To be independent on the PM system, it is assumed that the primary field $\mathbf{B}_0$ is given on regular grid of points located in the rectangular window $W = \{(x, y) : w_x \times w_y\}$ corresponding to the top and the bottom surfaces of the moving conductive block. The fields $\mathbf{B}^u_0(x, y, 0)$ and $\mathbf{B}^d_0(x, y, -l_z)$ in the window W can be calculated by any method e.g. FEM. $\mathbf{B}_0$ and φ_0 at any point $\mathbf{P} \in \Omega$, $\Omega = \{(x, y, z) : W \times < -l_z, 0 >\}$ are determined by solving $\nabla^2\mathbf{B}_0 = \mathbf{0}$ and $\nabla^2\varphi_0 = 0$ with the boundary conditions shown in Fig. 2. The solution is obtained by applying 2D spatial Fourier transform: $\tilde{\mathbf{B}}_0(z) = \mathcal{F}_x\mathcal{F}_y(\mathbf{B}_0)$ and $\tilde{\Phi}_0(z) = \mathcal{F}_x\mathcal{F}_y(\varphi_0)$. Having formulas for $\tilde{\mathbf{B}}_0$ and $\tilde{\Phi}_0$, the fields ($\mathbf{B}_0$, φ_0) at any plane $z \in < -l_z, 0 >$ can be calculated using the inverse Fourier transform $\mathbf{B}_0(\mathbf{r}) = \mathcal{F}_x^{-1}\mathcal{F}_y^{-1}(\tilde{\mathbf{B}}_0)$ and $\varphi_0(\mathbf{r}) = \mathcal{F}_x^{-1}\mathcal{F}_y^{-1}(\tilde{\Phi}_0)$.

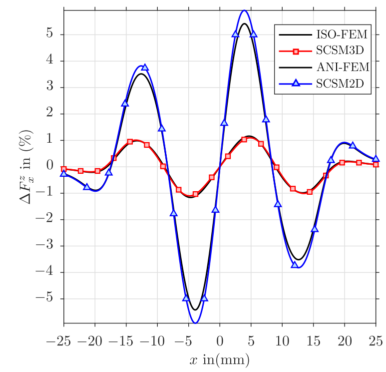

Fig. 3. Defect response signals ΔF^z_x calculated by FEM and the SCSM in the vicinity of the defect $d_x \times d_y \times d_z = 12\,\mathrm{mm} \times 2\,\mathrm{mm} \times 2\,\mathrm{mm}$ located at a depth $d = 2\,\mathrm{mm}$ in the aluminum block $l_x \times l_y \times l_z = 50\,\mathrm{mm} \times 25\,\mathrm{mm} \times 10\,\mathrm{mm}$.

III. Results

To verify the SCSM, the following LET problem is solved using FEM: the aluminum block of isotropic/anisotropic conductivity $\sigma_0 = 21\,\mathrm{MS/m}$ with the ideal defect ($\sigma_d = 0$) moves with the velocity $v = 0.2\,\mathrm{m/s}$ under the magnet system located at the lift-off distance of $h = 1\,\mathrm{mm}$. Fig. 3 shows the defect response signals (DRS) defined as

$$\Delta F^z_x = \left(\frac{F_z}{F_x} - \frac{F_{0z}}{F_{0x}} \right) 100\% \;\longleftarrow\; \mathbf{F} = \int_{\Omega_c} \mathbf{J} \times \mathbf{B}_0 \, \mathrm{d}V, \tag{6}$$

calculated by FEM and the SCSM. It can be observed that the DRS for the block with anisotropic conductivity is about 5 times bigger than for the block with isotropic conductivity. The quantitative differences between FEM and the SCSM results, expressed by the normalized root mean square error, are equal $\epsilon_1 = 1.91\%$ and $\epsilon_2 = 2.25\%$, for the isotropic and anisotropic block, respectively.

IV. Conclusion

In the full paper, the comparison of the SCSM and FEM will be extended and discussed. Moreover, the results will be validated with measurements.

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